1.1 The Concept of Instantaneous Rate of Change Homework

Name Key

Date ______ Period ______

1. An object dropped from a state of rest at time $t = 0$ travels a distance $s(t) = 4.9t^2$ meters in t seconds.

   A. How far does the object travel during the time interval [2.5, 3].

   $\Delta S = S(3) - S(2.5) = 44.1 - 30.625 = \boxed{13.475 \text{ meters}}$

   B. Find the average velocity of the object over [2.5, 3].

   $\text{AROC} = \frac{S(3) - S(2.5)}{3 - 2.5} = \boxed{26.95 \text{ m/sec}}$

   C. Estimate the object's instantaneous velocity at $t = 2.5$ seconds.

Interval	[2.5, 2.51]	[2.5, 2.505]	[2.5, 2.5001]
Average Velocity	24.5	24.6	20

IROC @ t = 2.5 sec is ≈ 20 m/sec

2. Suppose Neil Armstrong decided to throw a golf ball into the air while he was standing on the moon and that the height of the golf ball was modeled by the equation below, where s is measured in feet and t is measured in seconds $s(t) = -2.72t^2 + 26.9t + 6$. Find the best approximation for the instantaneous rate of change (velocity) of the golf ball at 7 seconds.

$\text{IROC} \approx \frac{S(7.0001) - S(7)}{7.0001 - 7} \to \frac{61.019 - 61.02}{.0001} \approx \boxed{-10 \text{ ft/sec}}$

3. A pendulum swings from the ceiling. Its distance, d, in feet, from one wall of the room depends on the number of seconds, t, since it was set in motion. Assume that the equation for d as a function of t is $d(t) = 20 + 16\cos\left(\frac{\pi}{3}t\right)$. You want to find out how fast the pendulum is moving at a given instant, t, and whether it is approaching or going away from the wall.

   A. Find d when $t = 4$. What mode should your calculator be set?

   $d(4) = 12$ ft Radian Mode

   B. Estimate the instantaneous rate of change of d with respect to t when $t = 2.5$. At that time, is the pendulum approaching the wall or moving away from it? Explain how you know.

$\text{IROC} \approx \frac{d(2.5001) - d(2.5)}{2.5001 - 2.5} \approx -8.00 \text{ ft/sec}$

Rate is negative, so distance is decreasing so Pendulum is approaching the wall.

4. Let $C(t)$ be the concentration of a drug in the bloodstream. As the body eliminates the drug, $C(t)$ decreases at a rate that is proportional to the amount of the drug that is present at the time. Write an algebraic expression to represent the average rate of change of the drug's concentration for the period $t = 0$ hour to $t = 4$ hours after the drug has been administered.

$$\frac{\Delta C}{\Delta t} = \frac{C(4)-C(0)}{4-0} \Rightarrow \boxed{\frac{C(4)-C(0)}{4}}$$

5. Compute $\Delta y/\Delta x$ for the interval [2,6], where $y = 4x - 7$. What is the instantaneous rate of change of y with respect to x at $x = 2$?

$$\frac{\Delta y}{\Delta x} = \frac{f(6)-f(2)}{6-2} = \frac{17-1}{4} = 4 \qquad \frac{f(2.001)-f(2)}{2.001-2} = 4$$

AROC = IROC = 4 @ X = 2

6. An initial deposit of \$500 in your bank will have a balance after t years given by the equation $P(t) = 500(1.06)^t$ dollars.

A. What are the units of the rate of change of $P(t)$? dollars/years

B. Find the average rate of change over [0, 1].

$$\text{AROC} = \frac{P(1)-P(0)}{1-0} = \frac{530-500}{1} = \$30/\text{year}$$

C. Estimate the instantaneous rate of change at $t = 1$ by computing the average rate of change over intervals to the left and right of $t = 1$.

$$\frac{P(1)-P(.999)}{1-.999} = \$30.00 \qquad \frac{P(1.001)-P(1)}{1.001-1} = \$30.00$$

$$\boxed{\text{ABOUT } \$30.00/\text{year}}$$

7. Each position graph shown below represents particle motion as a function of time. Match each graph with the proper description:

A) Speeding up **B)** Speeding up and then slowing down

C) Slowing down **D)** Slowing down and then speeding up